A profitable trading and risk management strategy despite transaction cost

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Abstract

We present a new profitable trading and risk management strategy with transaction cost for an adaptive equally weighted portfolio. Moreover, we implement a rule-based expert system for the daily financial decision making process by using the power of spectral analysis. We use several key components such as principal component analysis, partitioning, memory in stock markets, percentile for relative standing, the first four normalized central moments, learning algorithm, switching among several investments positions consisting of short stock market, long stock market and money market with real risk-free rates. We find that it is possible to beat the proxy for equity market without short selling for 168 S&P 500-listed stocks during the 1998-2008 period and 213 Russell 2000-listed stocks during the 1995-2007 period. Our Monte Carlo simulation over both the various set of stocks and the interval of time confirms our findings.

Keywords portfolio risk management, algorithmic trading, out-of-sample prediction, long memory in stocks, adaptive learning algorithm, market timing, principal component analysis, simulation, volatility, behavioral finance

AMS subject classifications 91G10, 91G60, 91B30, 62M10, 62M15, 62M20, 65F15, 62C12, 05A18, 65C05

JEL subject classifications G11, C6, D83, D81, G14

1 Introduction

Algorithmic trading and risk management are significant topics in the investment literature. We propose a new algorithmic approach that uses spectral properties of empirical stock correlation matrices related to random matrix theory and multivariate analysis (see Hardle and Simar [15], Marchenko and Pastur [19], Mehta [21], and Wishart [26]) to develop a new time-dependence model for profitable portfolio risk management in presence of real risk-free rate and transaction cost.

Several studies (see Laloux et al. [17], Kim and Jeong [16], and references therein) considered financial correlation matrices with significant contributions. Laloux et al. [17] argue that while the

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bulk eigenvalues are in agreements with the random correlation matrix, Markowitz's optimization scheme [20] is not sufficient because the lowest eigenvalues of the historical correlation matrix are dominated by noise and they determine the smallest risk portfolio (see Appendix). It is worthwhile to focus on large eigenvalues to extract clear information for volatility. Kim and Jeong [16] decompose the correlation matrix into market, group (sector), and the Wishart random bulk (noise terms).

It is very challenging to find a profitable investment strategy in the stock market. Caginalp and Laurent [5] did the first successful scientific test for this purpose by following out-of-sample procedure on a large scale data set. They observed statistically significant profit of almost 1% during a two-day holding period, for stocks in the S&P 500, between 1992 and 1996, by using non-parametric statistical test for the predictive capabilities of candlestick patterns. Blume et al. [1] consider technical analysis as a component of agents' learning process. They argue that sequences of volume and price can be informative by examining the informational role of volume closely. Moreover, they argue that traders who use information contained in the market statistics attain a competitive advantage. Later, Caginalp and Balenovich [4] present a theoretical foundation for technical analysis of securities by employing a nonlinear dynamical microeconomic model. They illustrate a wide spectrum of patterns that are generated by the presence of multiple (heterogeneous) investor groups with asymmetric information, besides the patterns for the trading preferences in a single group. Shen [24] presents a market timing strategy based on the spread between the E/P ratio of the S&P 500 index and a short-term interest rate. The strategy beats the market index with monthly data from 1970 to 2000 even when transaction costs are incorporated. Furthermore, Rapach et al. [23] study international stock return predictability with macro variables by using in-sample and out-of-sample procedures with data mining.

Duran and Caginalp [8] define a deviation model and suggest an algorithm that can be useful for prediction of various stages of financial overreaction and bubbles. Moreover, Duran and Caginalp [9] find a characteristic overreaction diamond pattern with statistically significant precursors and aftershocks for significant price changes by accomplishing noise elimination via their deviation model. More recently, Duran [7] and Duran and Caginalp [10] study an inverse problem involving a semi-unconstrained parameter optimization for a dynamical system of nonlinear asset flow differential equations to describe trader population dynamics. They develop a semi-dynamic multi-start approach and present the corresponding asset flow optimization forecast algorithm. The empirical results for a number of closed-end funds trading in US markets show that their out-of-sample prediction beats the default theory of random walk, by applying non-parametric tests consisting of the Mann-Whitney U test and Wilcoxon rank sum test.

Artificial intelligence has been developing since the middle of the 20th century and it has been widely used for stock trading. However, White [25] showed that using neural networks with 500 days of IBM stock was unsuccessful in terms of short term forecasts. More recently, Pan et al. [22] discussed seven potential components of intelligent finance. Additionally, we believe that expert systems (see Feigenbaum [13]), knowledge-based computer programs with a set of inference rules ('if then' type of statements) in a rulebase, are among the most promising subfields in artificial intelligence for stock return forecasting. We use an expert system with forward chaining as a reasoning method to reach conclusions in our learning algorithm.

Existence of long memory in financial markets is an important topic that has been of considerable interest to researchers. Ederington and Guan [11] develop a new volatility forecasting model and find that financial markets have longer memories than obtained in GARCH(1,1) model estimates. However, they observe that this has made little progress in out-of sample forecasting. Moreover, the long-memory of supply and demand has been discussed in several papers ((see Lillo

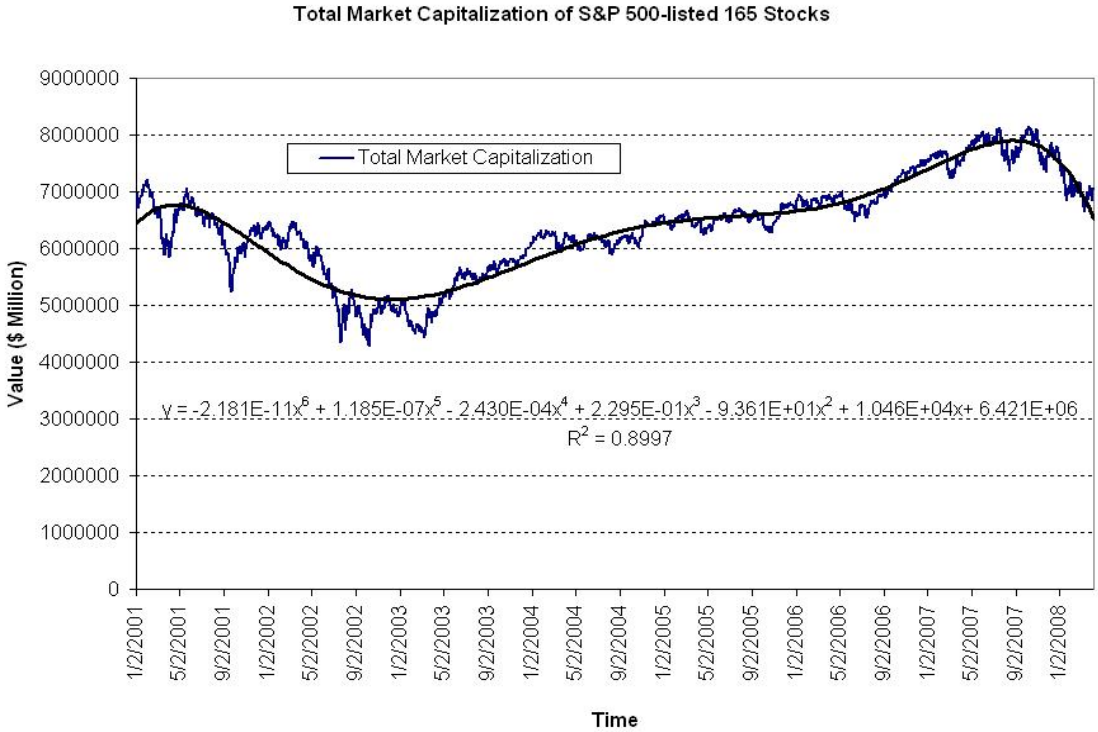

Figure 1: Total market capitalization for S&P 500-listed 165 stocks.

and Farmer [18], Bouchaud et al. [3], Farmer et al. [12], and references therein for the fruitful discussions). In this paper, we try to explore whether there is a relationship between long memory and volatility for S&P 500-listed stocks and Russell 2000-listed stocks. If so, can we use it for a profitable investment strategy?

Figure 1, shows that the total market capitalization for S&P 500-listed 165 stocks ranges between 4.3 and 8.1 trillion dollars during the 2001-2007 period. We apply regression analysis and find that it is approximated by a sixth order polynomial $y = -2.181E-11x^6 + 1.185E-07x^5 - 2.430E-04x^4 + 2.295E-01x^3 - 9.361E+01x^2 + 1.046E+04x + 6.421E+06$ which explains 89.97% of the variation for this time interval. The polynomial has both convex and concave down curve segments. Moreover, the total market capitalization for the S&P 500-listed 165 stocks in January 2007 is approximately 15% of the total market capitalization of all publicly traded companies in the world which is approximately US$51.2 trillion in January 2007 according to Reuters. While most of 168 S&P 500-listed stocks are large-cap companies, the portfolio of 213 Russell 2000-listed stocks consists of relatively small-cap companies. Thus, the portfolio of S&P 500-listed stocks and the portfolio of Russell 2000-listed stocks are good proxies for equity market.

Many authors have documented that they couldn't find an arbitrage strategy (see Capinski and Zastawniak [6], Chapter 4 for the definition of No-Arbitrage Principle) in a market with transaction costs. The proposed algorithmic trading strategy is an illustrative counter example by using daily dividend adjusted closing prices. Based upon the signals given by the current sample, a buy/sell/hold decision is made with respect to the market portfolio versus the risk-free asset. The strategy's performance is computed over a sample period of roughly nine and one half years for S&P 500 and twelve and half years for Russell 2000. Following the strategy soundly beats the proxy for S&P 500 and the proxy for Russell 2000 in both mean (higher) and in standard deviation (lower). The weak form of the efficient market hypothesis (EMH) (see Bodie et al. [2], Chapter 12 for the definition) asserts that this is not possible on any time scale, let alone even one day.

Two screens form the basis for our strategy's trades. The first screen is covariance based in that if the maximum eigenvalue of the sample assets' correlation matrix (over the trailing 100 trading days) is too high or too low, then the strategy is warned against the market. While a high volatility of market corresponds to panic, low volatility of market reminds silence before a storm in the market. Many investor groups would like to sell their shares for various reasons. There may be a temporary silence at the beginning of a credit crunch especially when prices are overvalued at a high level. Depending on whether short sales are allowed or not, either a short position or the risk-free asset is chosen for the next day. The second screen is applied when the maximum eigenvalue is medium. In this case, the 100-day history is used to calculate the first four sample central moments of each asset's returns because these four moments are more informative than the first two moments. Cross-sectional averages of four moments are then classified as either High, Medium, or Low depending on whether they fall in the top, middle, or bottom third of all recorded historical values. These three classifications are used to define a state vector for the market, i.e., a four dimensional vector with three possible values resulting in 81 possible market states. There is a trade of for the number of possible market states. While more partitions can provide more information, it is hard to find enough number of historical samples for each partition. Here, 81 is a feasible heuristic value. The state-conditional average of the ratio of the first two cross-sectional average moments μ/σ^2 is then considered. Are conditions favorable in a risk versus reward sense? If it is above unity, then the next day is a long position in the market. If it is below unity, then the next day is short the market (or risk-free asset if short selling is restricted). If it equals unity or if there is no history yet for the appropriate market state, then the risk-free asset is chosen.

We are not willing to change our position often and we try to minimize transaction cost. We prefer $-1 < \mu/\sigma^2 < 1$ instead of $-1 < \mu/\sigma < 1$ to choose risk-free investment because the interval for μ in the former case is narrower than that of the latter. Long term expected stock return is higher than that of risk-free investment. Thus, we may stay in stock market longer than that a typical risk-reward approach suggests in terms of standard deviation. On the other hand, we prefer risk-free investment to be on safe side when there is a persistence in agreement that current relative risk is extreme. We utilize the maximum eigenvalue approach as our dominant decision parameter for this.

With respect to market memory, we believe that many CEO's, analysts, investors and other participants focus very much on quarterly revenue growth and quarterly earnings report. Sometimes they may dominate decision making based on quarterly earning announcements rather than other objectives, factors or time intervals. Moreover, there may be several days between quarterly earning announcements for different companies. While each company prepares its own report with three months projection, it should interpret both the consequences of its previous quarterly report and the related companies' reports. Furthermore, it is important to determine the reasonable time to place your order to buy or sell. You may need to wait several days. Overall, they remind us a decision making process/focusing process which may take approximately 100 trading days. Therefore, we consider it as a heuristic memory value for the correlation matrix and the first four sample central moments.

The overall return for the proposed strategy is 572.62% in a period where the proxy of market gained 179.19% with the "buy-and-hold" strategy. The associated standard deviations of daily log returns are 0.78% and 1.32%, respectively. The sample assets are 168 members of the S&P 500. Each day after the close, a new correlation matrix is computed using the newest 100-day historical window. What changes over time, and where any learning occurs, is in the overall asset

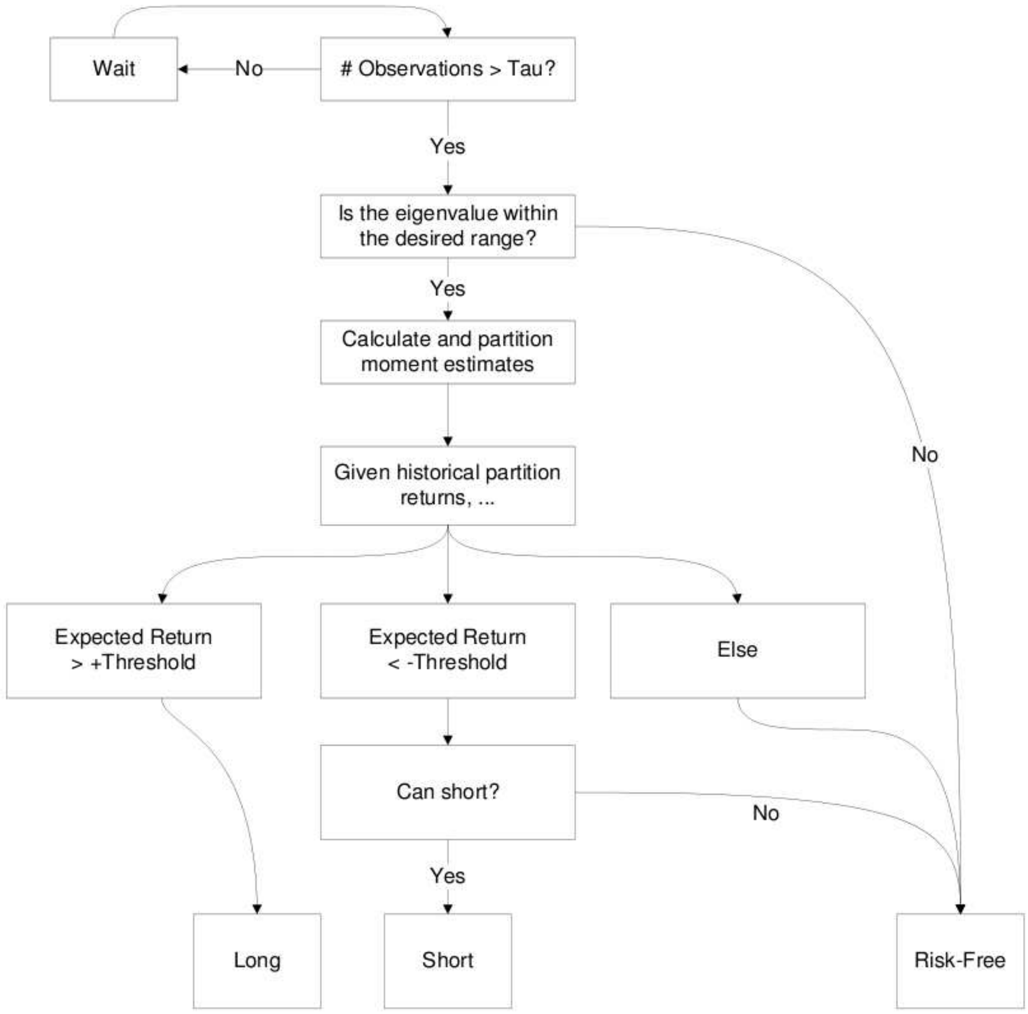

Figure 2: Flowchart for the summary of the daily financial decision making algorithm.

histories which define the market state vector as well as the two-directional threshold, for example 35% and 77%, for the eigenvalue screen.

The remainder of the paper is organized as follows. In Section 2, we present our algorithm for out-of-sample strategy. In Section 3, empirical results are illustrated by several examples and Monte Carlo simulations. The performance of the proposed strategy without short selling is compared with that of the proxy for stock market in terms of risk and return by using daily dividend adjusted closing prices for 168 S&P 500-listed stocks and 213 Russell 2000-listed stocks [1]. Section 4 concludes the paper. Appendix contains a summary for random matrix theory and Markowitz's optimization scheme.

[1]The list of stocks can be provided upon requested. For the S&P 500, we focused on 168 stocks because they are traded over the same dates and time range between April 29, 1998 and April 4, 2008 without being delisted/removed and became accessible via finance.yahoo.com. In order to test the performance of the algorithm on smaller capitalization stocks, we use the Russell 2000 index as a guide. Among the stocks that comprised the index in July 2009, we consider those stocks that were publicly traded between 1995 and 2007. This choice results in a data set of 213 small-cap stocks from 1995 to 2007. These are the only selection criteria.

2 Algorithm for out-of-sample strategy

We use a data set with N assets and $M+1$ observed daily closing prices. From these $M+1$ prices, we calculate the log-return vector for each asset and form the $M \times N$ log-return matrix R. We choose a value of τ, which determines the "memory" or "window" of each parameter's calculation.

$$R_{i+1,j} = log(P_{i+1,j}) - log(P_{i,j}), \quad i \ = \ 1, ..., M, \ j = 1, ..., N$$

We find the relative standing of the most recent window's four parameters consisting of mean, standard deviation, skewness, and kurtosis with respect to the current set of historical observations over windows up to yesterday. Generally, kurtosis is used to measure a high peak and heavy tails qualitatively (see Glasserman [14]). We choose a number of partitions for the percentile categorization. The number of partitions represents the 'fineness' of our categorization. That is, two partitions implies each parameter dimension has two states (above or below 50th percentile). We prefer percentile rather than z-score as a measure of the relative standing, because the percentile approach is a non-parametric test and it does not make any assumptions regarding the underlying probability distribution.

Let $C^\tau(t)$ be the time-dependent correlation coefficient matrix over the return matrix R. That is, define $C^\tau(t)$ such that, for $t \ \in \tau+2, \dots, M-1, M$

$$C^\tau_{i,j}(t) = \frac{\mathbf{E}[R_{(t-\tau+1,\dots,t),i} \quad R_{(t-\tau+1,\dots,t),j}] - \mathbf{E}[R_{(t-\tau+1,\dots,t),i}] \ \mathbf{E}[R_{(t-\tau+1,\dots,t),j}]}{\sqrt{\mathbf{E}[(R_{(t-\tau+1,\dots,t),i} - \mathbf{E}[R_{(t-\tau+1,\dots,t),i}])^2]}\sqrt{\mathbf{E}[(R_{(t-\tau+1,\dots,t),j} - \mathbf{E}[R_{(t-\tau+1,\dots,t),j}])^2]}}$$

Let $\Lambda^\tau(t)$ be the time-dependent maximum eigenvalue function. In other words, define $\Lambda^\tau(t)$ such that

$$\Lambda^\tau(t) = \frac{\mathbf{max}_i\{\lambda_i\}}{\sum_i \lambda_i}$$

From principal component analysis, we have that the sum of the eigenvalues of a matrix A is equal to the trace of A, where $trace(A) = \sum_i A_{i,i}$. Since $C^\tau(t)$ is a correlation matrix, these diagonal entries $C^\tau_{i,i}$ are all equal to 1 by definition of the correlation, so that we have trace $(C^\tau) = \sum_{i=1}^N 1 = N$. Thus, $\Lambda^\tau(t)$ may be rewritten as $\Lambda^\tau(t) = \frac{max_i\{\lambda_i\}}{N}$. $\Lambda^\tau(t)$ is thus the ratio of the maximum eigenvalue to the total number of assets and corresponds to the real-time maximum proportion of the variance. This normalization to the proportion allows $\Lambda^\tau(t)$ to be compared between markets with different numbers of assets. τ is the memory of the correlation matrix, determining the number of previous periods to include in calculations.

We use buy and hold strategy for the proxy which uses equally weighted portfolio initially at the beginning of investment. The weights change in time as stock prices change.

We use equally weighted portfolio initially for the strategy. We keep the number of shares fixed unless there is a switch. When there is a switch to long, we find new number of shares based on new equally weighted portfolio, wealth and share prices. See Figure 9 and Figure 19 for the time-series of fund allocation using the strategy for the S&P 500-listed stocks and Russell 2000-listed stocks respectively.

We compute the logarithmic return k_V on the strategy's portfolio of N stocks in terms of the logarithmic returns $R_{i+1,j}$ at time $i+1$ by using the following formula.

$$k_V(i+1) = log(\sum_{j=1}^N w_j e^{R_{i+1,j}}).$$

It is different from the weighted average of individual logarithmic returns. Our strategy is a self-financing investment strategy. The algorithm for the out-of-sample strategy is as follows.
Algorithm 1.

1. For the first $\tau+1$ periods, we have fewer observed returns than that our window size requires. Thus, no calculation is possible.

2. For each day after $\tau + 1$ periods have been observed, perform the following

   (a) Calculate the maximum eigenvalue function value of the correlation coefficient matrix over the past τ returns including today t.

   (b) If the maximum eigenvalue function value of the current window exceeds the β-percentile or is below α-percentile of the maximum eigenvalue function values over all available historical data up to yesterday $t - 1$, invest in the risk free asset. For example, out of the range of the interval $[35\%, 77\%]$.

   (c) Otherwise, for each asset, calculate and store mean return, standard deviation of return, skewness of return, and kurtosis of return over the past τ returns.

   (d) Calculate the average over all assets for mean return, standard deviation of return, skewness of return, and kurtosis of return.

   (e) Determine which percentile partition each parameter falls in based on the historical values. Each partition number represents a different dimension of the state vector.

   (f) Store the average return over N securities as linked to the current state.

   (g) Given current state vector, search for the same historical states.

   - If there is no historical information about the current window parameters, then choose risk-free investment for tomorrow.
   - If we have historical information about current state, then compute the mean value of the historical returns per variance ($\frac{\mu}{\sigma^2}$) with the same state as today and make a decision for tomorrow:

     i. If $\frac{\mu}{\sigma^2} > 1$, go long on the stock market.

     ii. If $\frac{\mu}{\sigma^2} < -1$, go short on the stock market.

     iii. Otherwise, choose risk-free investment.

This strategy may be represented by a flowchart in Figure 2 where the threshold is σ^2.

3 Empirical results

We present our findings with Algorithm 1 using experience (learning) by the following examples. We try to observe the impact of each tool in the algorithm on the performance of strategy incrementally. First, we consider the strategy without using maximum eigenvalues. Later, we add the use of two-directional maximum eigenvalue.

3.1 Categorization with the first four moments

We analyze the performance of the percentile categorization with the first four moments by using 168 S&P 500-listed stocks and 213 Russell 2000-listed stocks. Moreover, we perform Monte-Carlo simulation over random subsets of both the set of stocks and the interval of time.

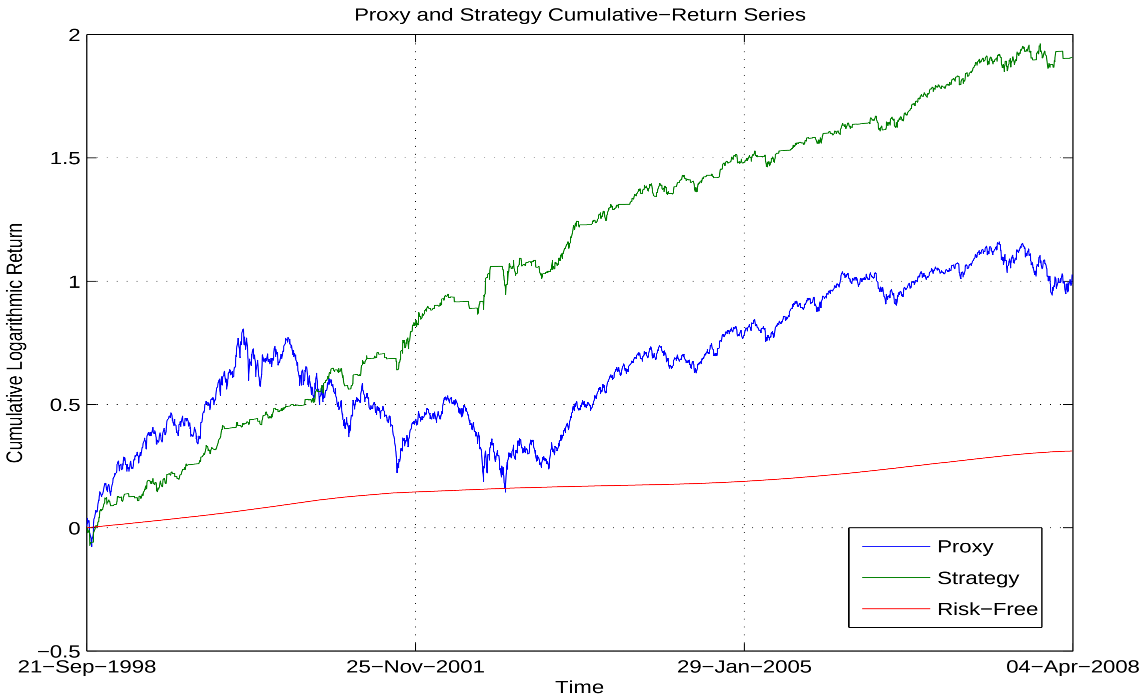

Figure 3: Cumulative logarithmic return for 168 S&P 500-listed stocks where $\tau = 100$, partitions $= 3$, and there is no short selling.

3.1.1 S&P 500-listed stocks

Given $\tau + 1$ daily adjusted closing prices of 168 S&P 500-listed stocks, we construct a portfolio obtained by either an equally weighted portfolio of the risky securities from S&P 500-listed stocks or completely current risk-free security and begin trading by following Algorithm 1.

We use daily data to make forecasts for the next day and have the overnight time interval from the close of trading to the open of the next day. Let the memory τ be 100 trading days and the number of partitions be 3 for the percentile categorization. Short selling is not allowed. Daily risk-free rates are used. The transaction cost incurred is 1% when we change our position. It will be based on tomorrow's closing price because we focus on out-of-sample prediction and we assume that we place our order to buy or sell immediately before the close of trading tomorrow. Of course we may use tomorrow's opening price or high frequency data in practice. We believe that the strategy will be more profitable then because of more flexibility and less delay. This issue will be discussed in our next paper.

We wait and collect data for 101 trading days between April 29, 1998 and September 20, 1998. We invest in risk-free or risky securities from September 21, 1998 to April 4, 2008. Figure 3 shows the performance comparison of a proxy for market with the "buy-and-hold" strategy, our strategy and risk-free account. The proxy of market can outperform our strategy during the warm up period, because we don't have enough experience in terms of historical information for new states that we addressed in the Introduction. In other words, there are 81 possible market states and it is hard to find a statistically significant number of historical samples for each partition. In business and economics, there is a break-even point where the revenue of an investment equals the cost of investment after a warm up period. Luckily, an investor may not trade prior to this break-even point, as they can collect and analyze data without taking a real position in the market.

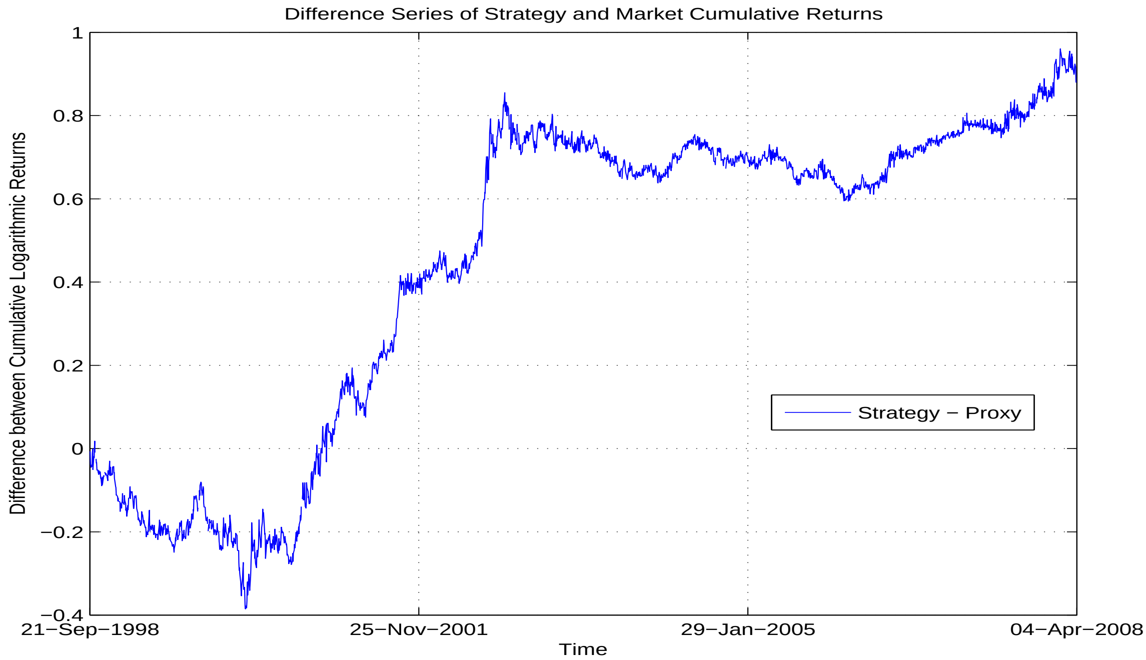

Figure 4: Time series of difference between cumulative logarithmic returns of the strategy and the proxy portfolio with 168 S&P 500-listed stocks without short selling.

Our strategy outperforms the proxy after the break-even point. Figure 4 displays the positive difference (after the warm up period) between the green and blue curves in Figure 3.

When to enter stock market and when to exit are very important factors for an arbitrage strategy. Investor can wait for a better position in stock market. We may have an idea approximately about possible outcomes for different initial points keeping in mind a learning warm up period, because logarithmic return has additivity property. The additivity property of logarithmic return allows us to consider many cases with arbitrary finite time intervals. In Figure 4, there are many increasing curve segments regarding arbitrage opportunities over time subintervals. That is, we have many time choices to place order to buy or sell so that we may beat the proxy. Figure 5 shows the cumulative classic return which is obtained from the logarithmic return. The strategy provides 3.2 times cumulative return as much as the proxy between September 21, 1998 and April 4, 2008 (see Table 1).

Figure 6 displays the time series of the average first four moments for 168 S&P 500-listed stocks related to part 2 (d) in Algorithm 1. We need each of the first four moments for more information.

We find that the number of desired sample returns filtered by partition has average 72.55 and standard deviation 48.99 in Figure 7. Generally there are more desired historical samples above the average after 2001. Moreover, we observe that there is a shortage of desired historical sample whenever there is a new big excitement in stock market.

Figure 8 shows the time series of mean value and variance of historical logarithmic return filtered by partition for the equally weighted portfolio, respectively. They become smoother in time with more desired historical samples that provide us with more statistically significant signals.

In Table 2, our strategy beats the proxy where the strategy has smaller risk and larger mean daily logarithmic return than that of the proxy. The kurtosis of the strategy is larger than that of the proxy mainly because of having higher peak rather than heavy tail.

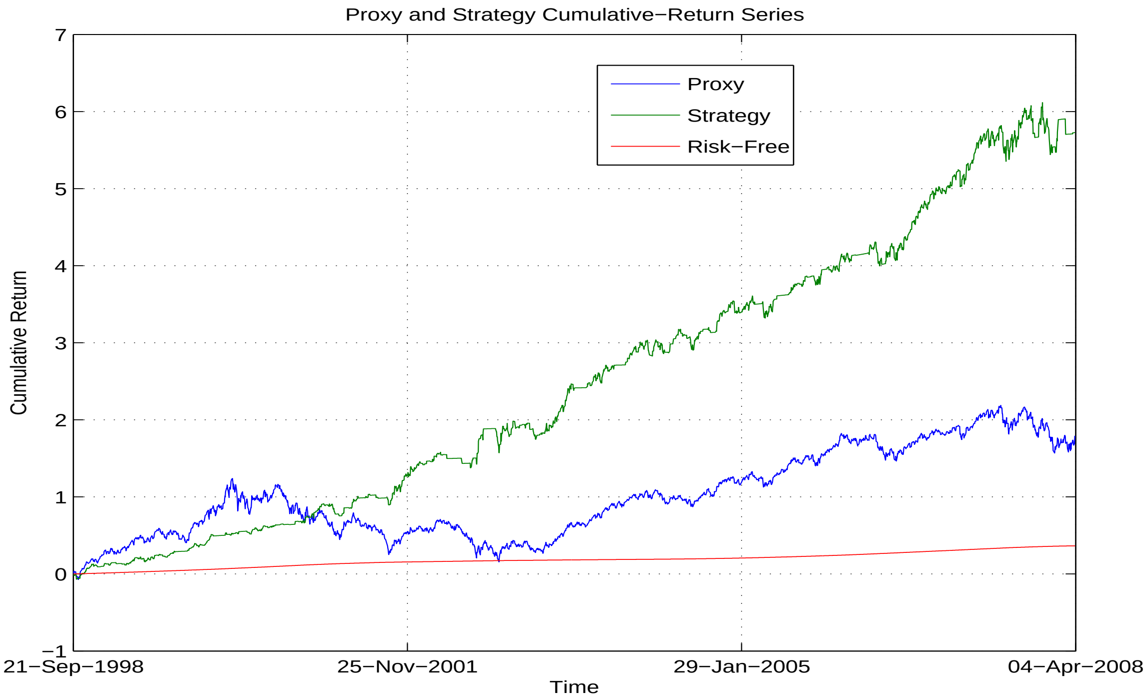

Figure 5: Cumulative classic return obtained via logarithmic return for 168 S&P 500-listed stocks where $\tau = 100$, partitions = 3, and short selling is not allowed.

Table 1: Performance comparison of the strategy, proxy of stock market, and money market in terms of average logarithmic and classic returns over the aggregate period between September 21, 1998 and April 4, 2008 where there is no short selling.

	Strategy	Proxy	Money Market
Logarithmic Return	190.60%	102.67%	31.09%
Classic Return	572.62%	179.19%	36.47%

Table 2: Performance comparison of the strategy, proxy of stock market, and money market in terms of daily logarithmic returns for 168 S&P 500-listed stocks without short selling.

	Strategy	Proxy	Money Market
Mean Daily Logarithmic Return	0.0802%	0.0432%	0.0131%
Standard Deviation	0.0078	0.0132	0.0001
Skewness	0.3732	0.0273	-0.1041
Kurtosis	10.0969	5.0708	1.5903

Figure 9 shows the time series of weights regarding the fund allocation of 168 S&P 500-listed stocks for the strategy. Figure 10 displays the time-series of partition states for four moments.

We observe that the average long position period is 1.66 times as long as the average risk-free one for the strategy without short selling, while the number of runs for the investment positions are equal to each other, in Table 3.

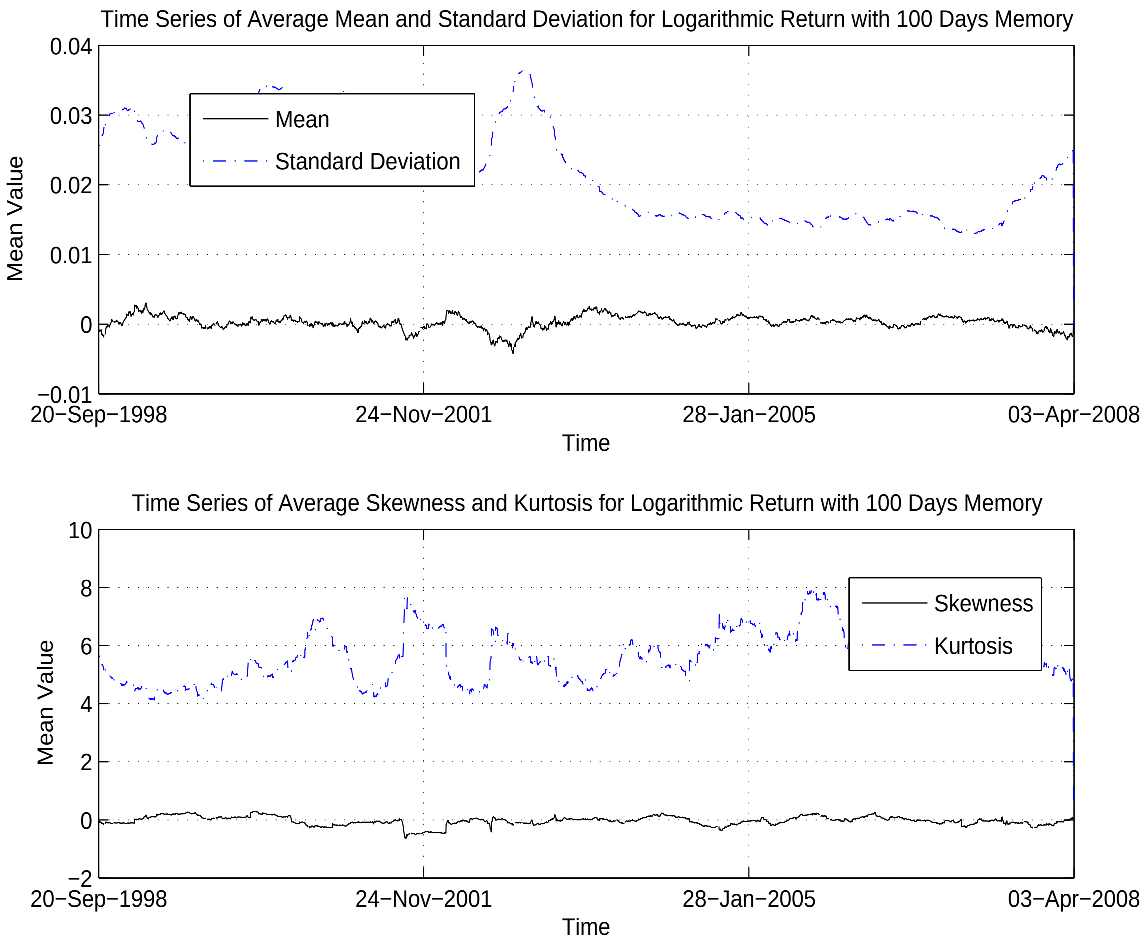

Figure 6: Time series for the average first four moments of logarithmic return for 168 S&P 500-listed stocks where $\tau = 100$.

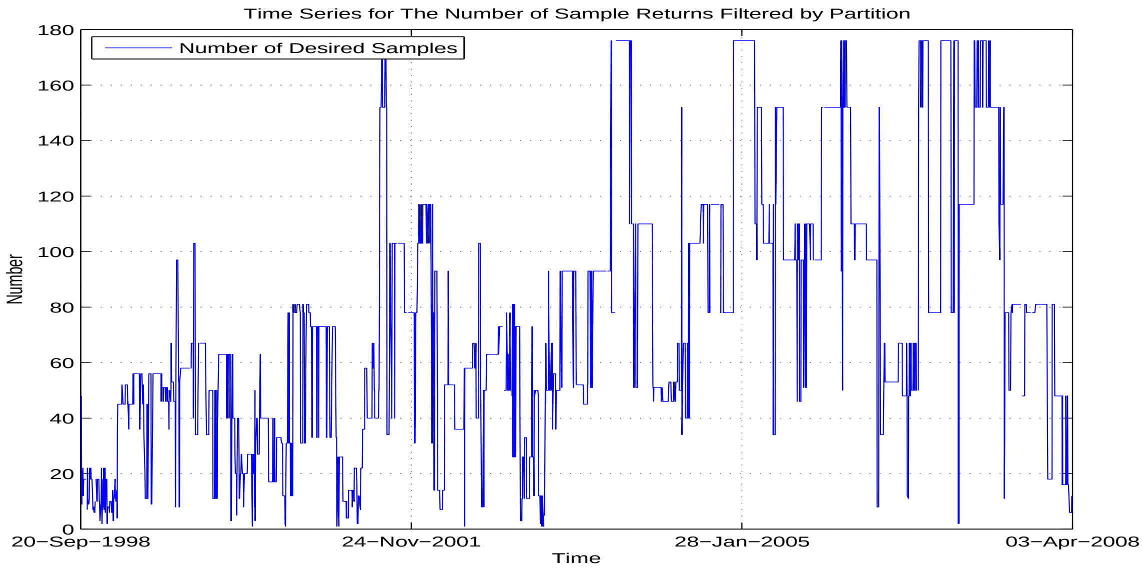

Figure 7: Time series for the number of desired sample returns from the equally weighted portfolio which consists of 168 S&P 500-listed stocks where $\tau = 100$ and partitions $= 3$.

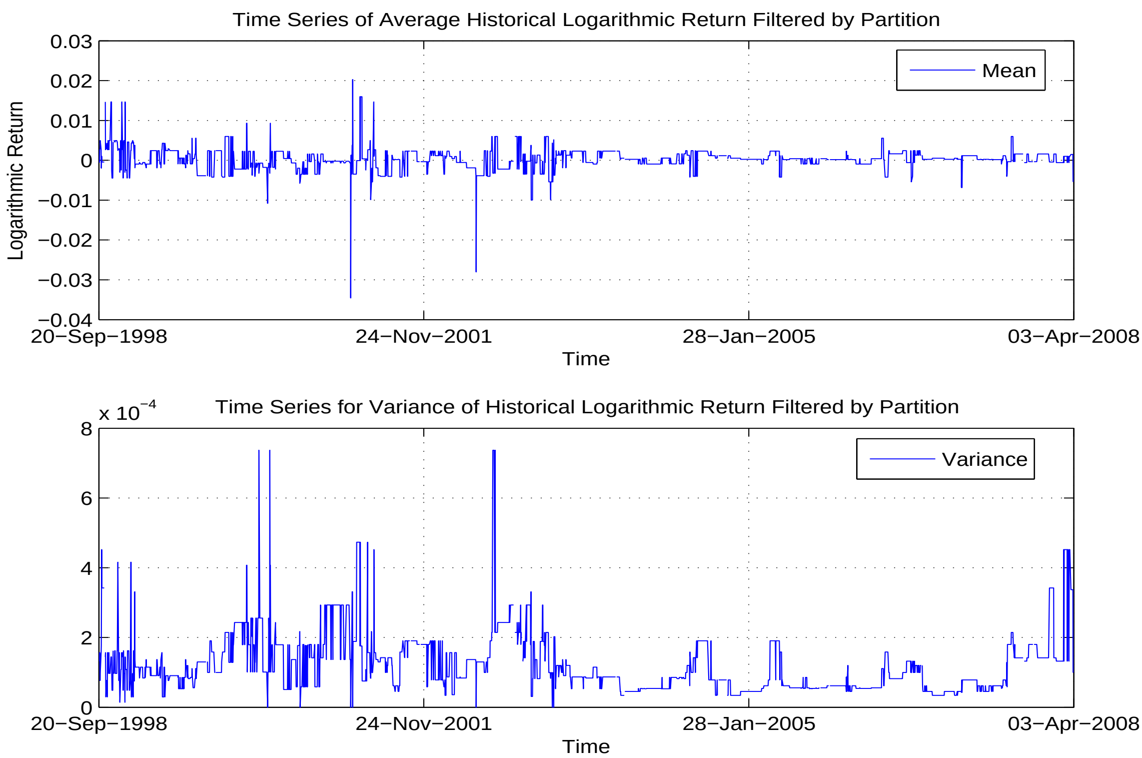

Figure 8: Time series of mean and variance of historical logarithmic return filtered by partition for the equally weighted portfolio which consists of 168 S&P 500-listed stocks where $\tau = 100$ and partitions $= 3$.

Table 3: Duration of investment positions and transaction cost for the strategy without short selling.

Long	1484 days
Risk-free	893 days
Average Long Period	7.77 days
Average Risk-free Period	4.68 days
Number of Long Runs	191
Number of Risk-free Runs	191
Number of Transaction Costs Incurred	382

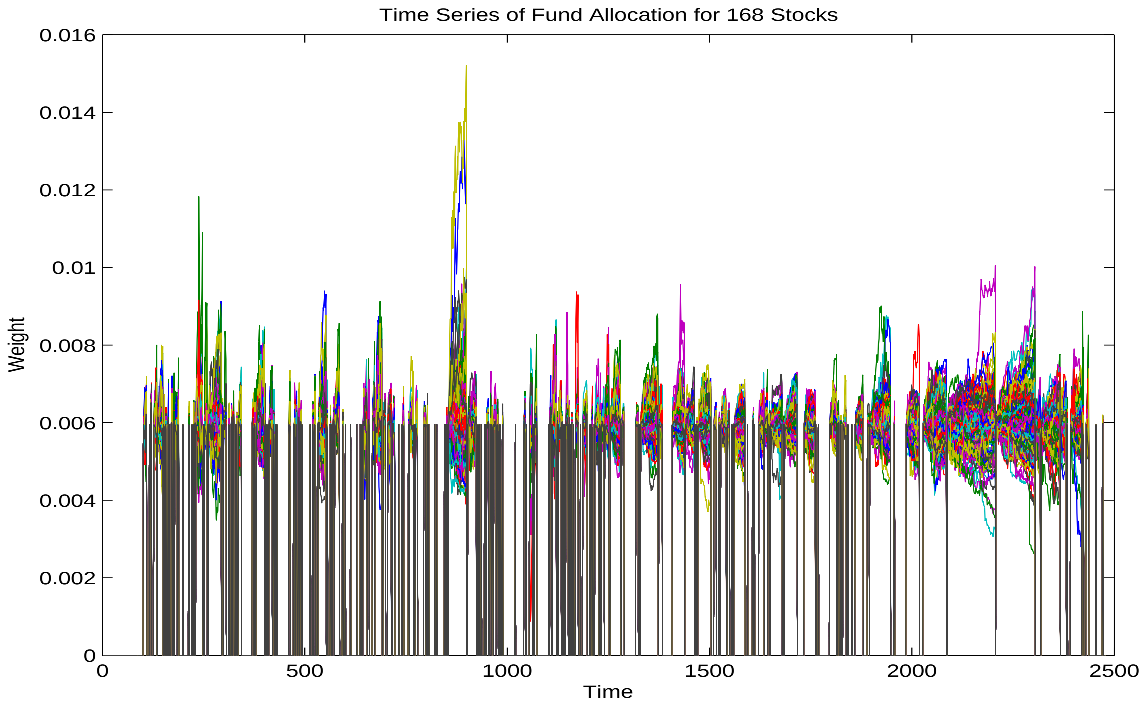

Figure 9: Fund allocation for 168 S&P 500-listed stocks where $\tau = 100$, partitions = 3, and short sales are not allowed.

3.1.2 Simulation for S&P 500-listed stocks

How sensitive is the success of our algorithm to the specific set of stocks and the specific chosen time interval? In order to verify the robustness of the algorithm, we performed Monte Carlo simulation over both the set of stocks and the interval of time. The data set is reduced to between 50 and 150 of the 168 stocks which are drawn at random in order to randomize the stocks. To randomize the time interval, a random offset of up to a year was removed from the beginning of the data set. Both of these randomizations occur for each iteration. For each iteration, the performance of the strategy and the proxy are compared in terms of the difference of mean log-returns as well as risk-adjusted return (units of return per unit risk). Figure 11 shows that the difference between average daily logarithmic returns of the strategy and the proxy converges to a positive value of 1.96E-4 based on 500 iterations. Figure 12 illustrates that the difference between average daily logarithmic returns per risk in terms of standard deviation for the strategy and the proxy converges to a positive value of 0.0425.

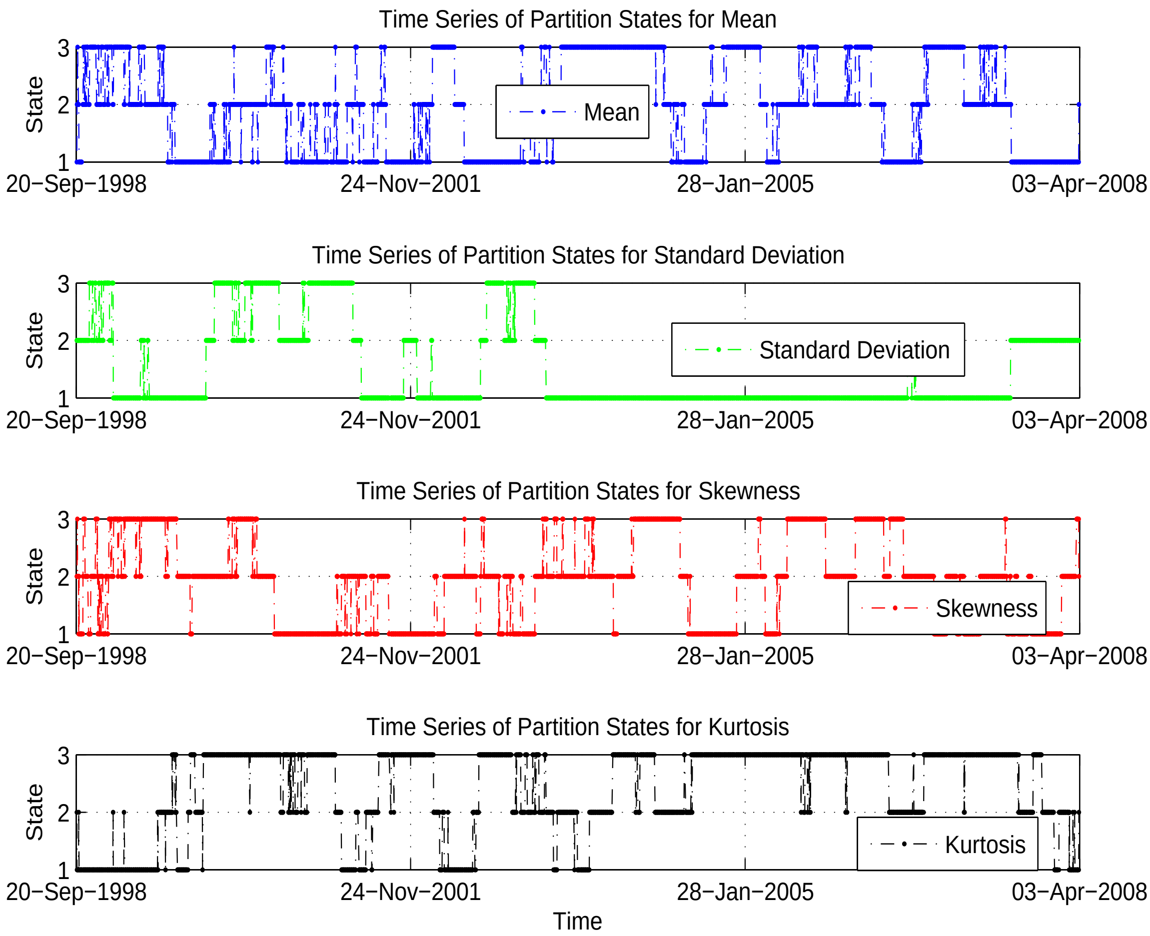

Figure 10: Time-series of partition states for four moments for 168 S&P 500-listed stocks where $\tau = 100$, partitions $= 3$, and short selling is not allowed.

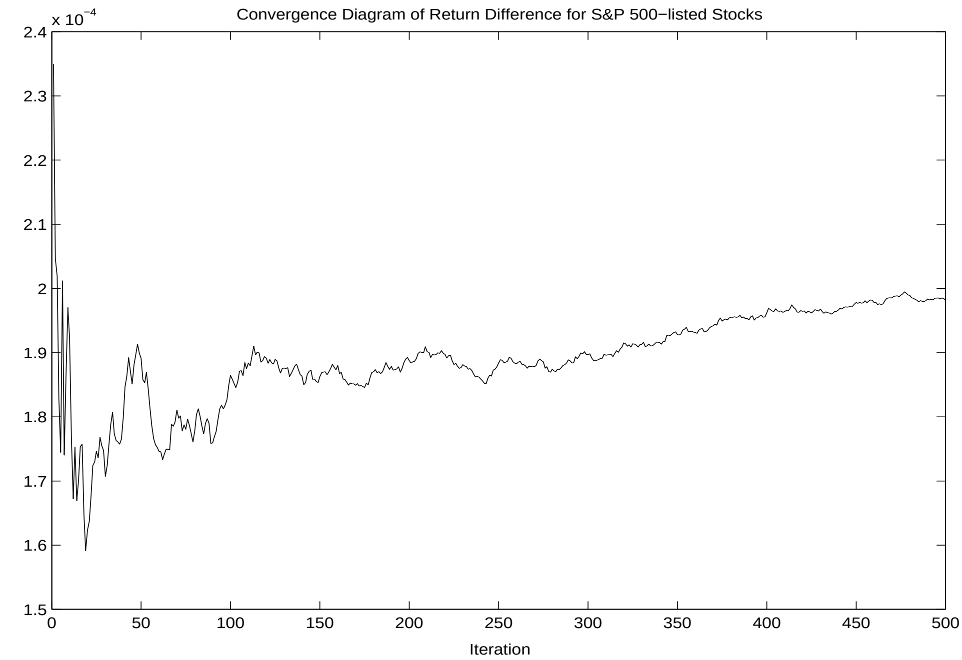

Figure 11: Monte Carlo simulation of the difference between average daily logarithmic returns of the strategy and the proxy for 168 S&P 500-listed stocks where $\tau = 100$, partitions = 3, and there is no short selling.

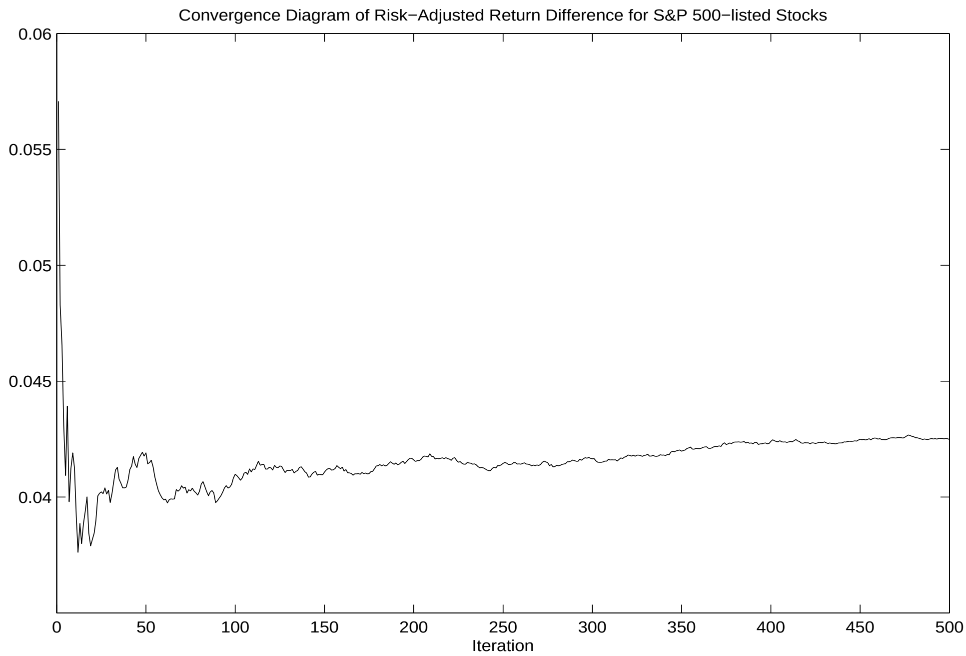

Figure 12: Monte Carlo simulation of risk-adjusted return difference for 168 S&P 500-listed stocks where $\tau = 100$, partitions = 3, and short selling is not allowed.

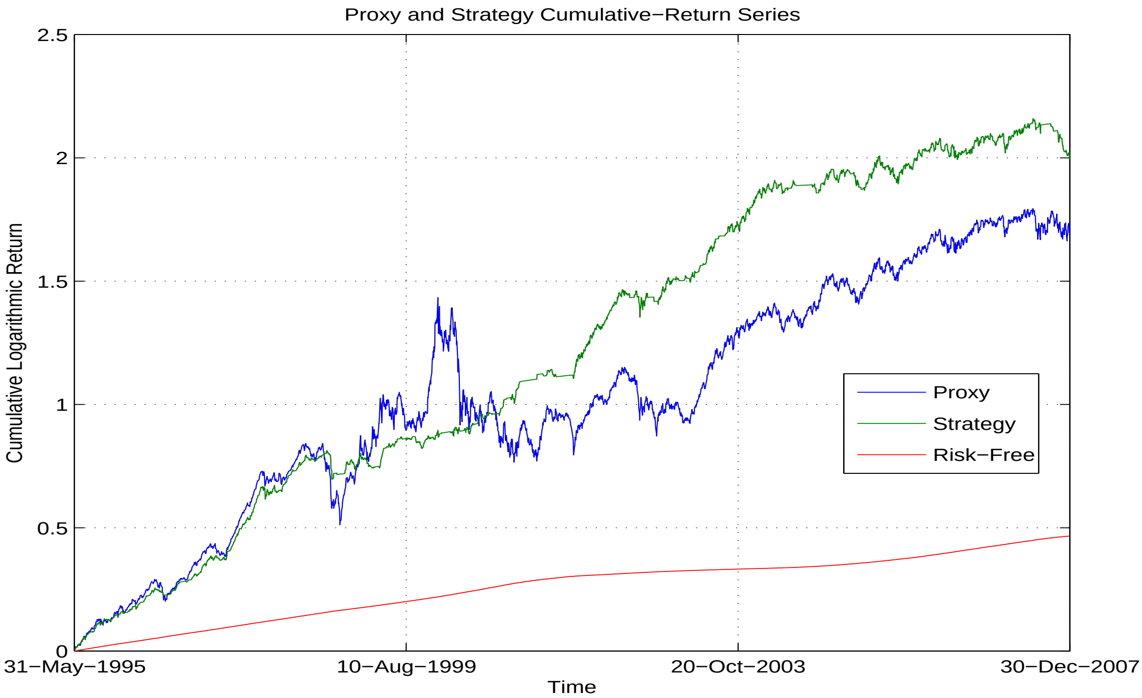

Figure 13: Cumulative logarithmic return for 213 Russell 2000-listed stocks without short selling.

3.1.3 Russell 2000-listed stocks

Our results for 213 Russell 2000-listed stocks are displayed in Figures 13-20 and Tables 4-6. Figure 19 displays the time series of weights regarding the fund allocation of 213 Russell 2000-listed stocks for the strategy. Figure 20 shows the time evolution of partition states for four moments.

Table 4: Performance comparison of the strategy, the proxy for Russell 2000 Index, and money market in terms of average logarithmic and classic returns over the aggregate period between May 31, 1995 and December 30, 2007 where there is no short selling.

	Strategy	Proxy	Money Market
Logarithmic Return	199.20%	169.12%	46.67%
Classic Return	633.05%	442.61%	59.47%

Table 5: Performance comparison of the strategy, the proxy for Russell 2000 Index, and money market in terms of daily logarithmic returns for 213 Russell 2000-listed stocks without short selling.

	Strategy	Proxy	Money Market
Mean Daily Logarithmic Return	0.0633%	0.0538%	0.0148%
Standard Deviation	0.0061	0.0127	0.0001
Skewness	-0.2876	0.0299	-0.6562
Kurtosis	8.6987	10.1655	1.9306

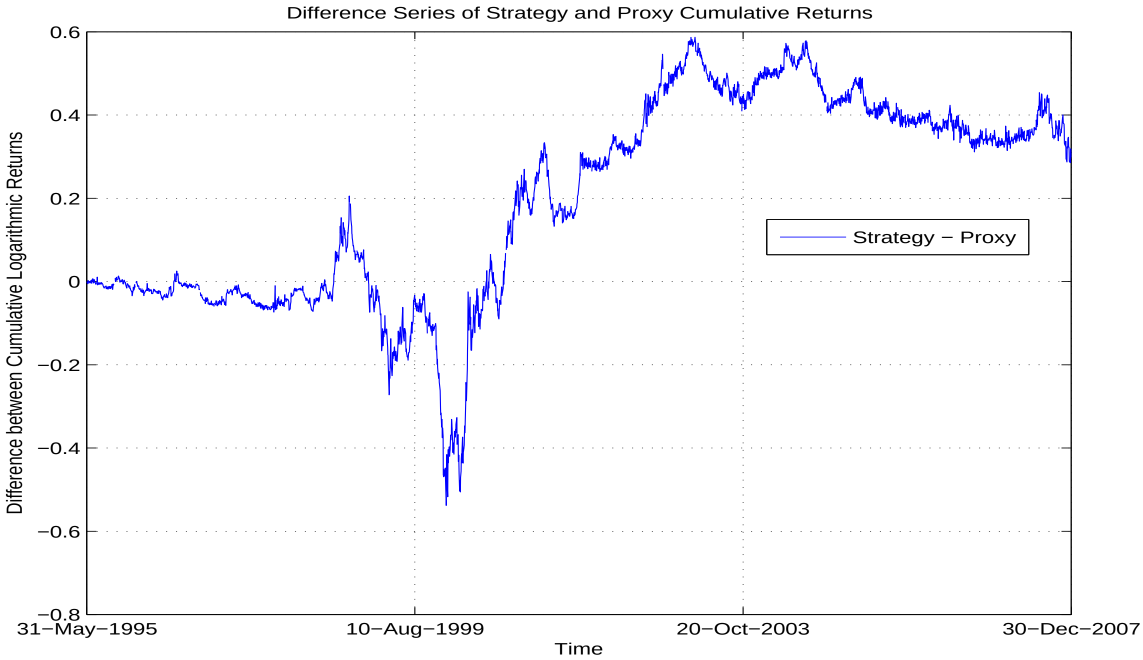

Figure 14: Time series of difference between cumulative logarithmic returns of the strategy and the proxy with Russell 2000 listed 213 stocks where $\tau = 100$, partitions $= 3$, and there is no short selling.

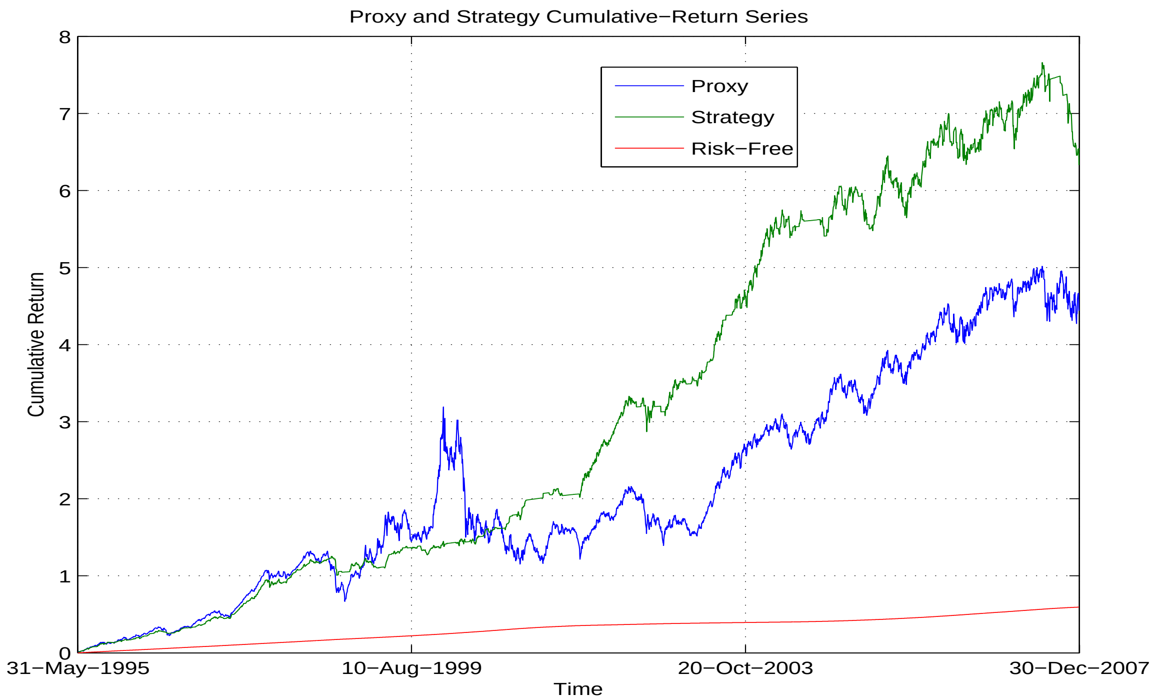

Figure 15: Cumulative classic return obtained via logarithmic return for 213 Russell 2000-listed stocks where $\tau = 100$, partitions $= 3$, and there is no short selling.

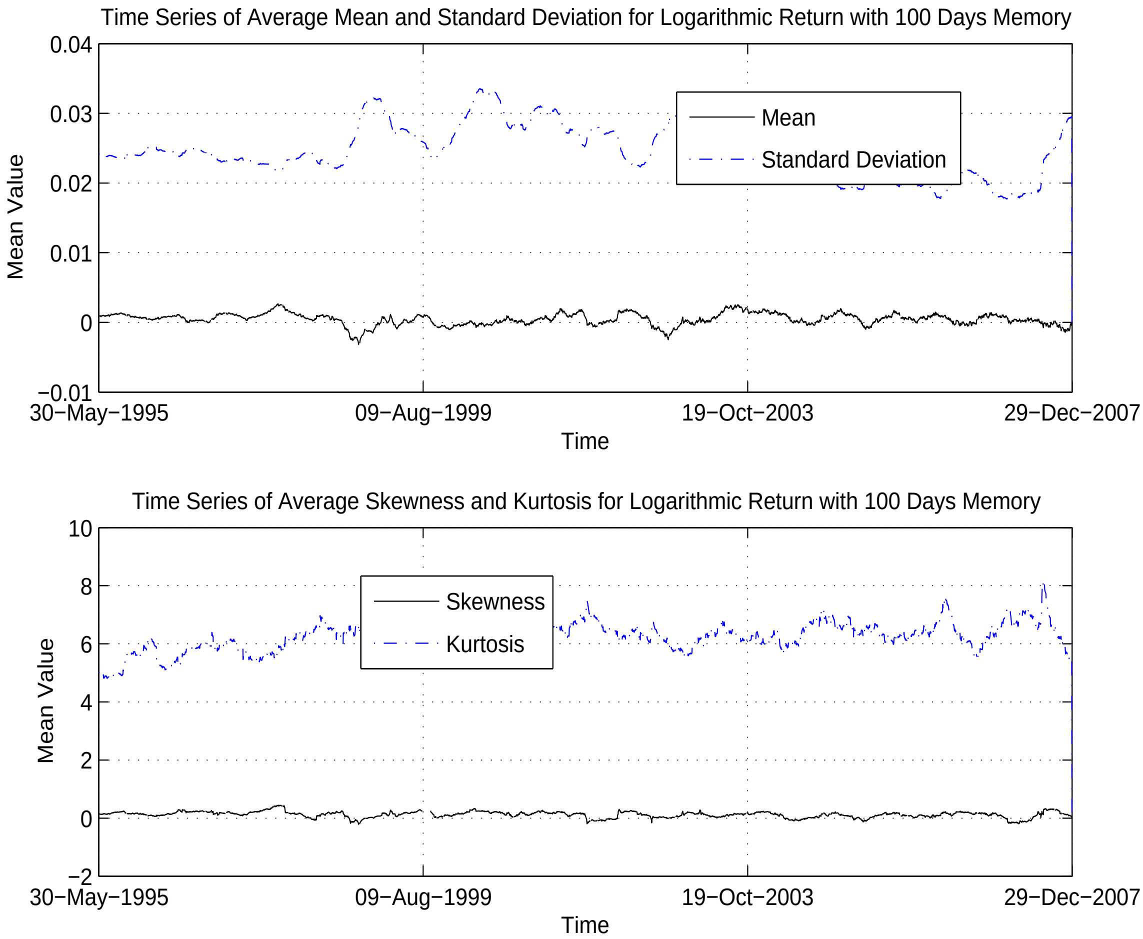

Figure 16: Time series for the average first four moments of logarithmic return for 213 Russell 2000-listed stocks where $\tau = 100$.

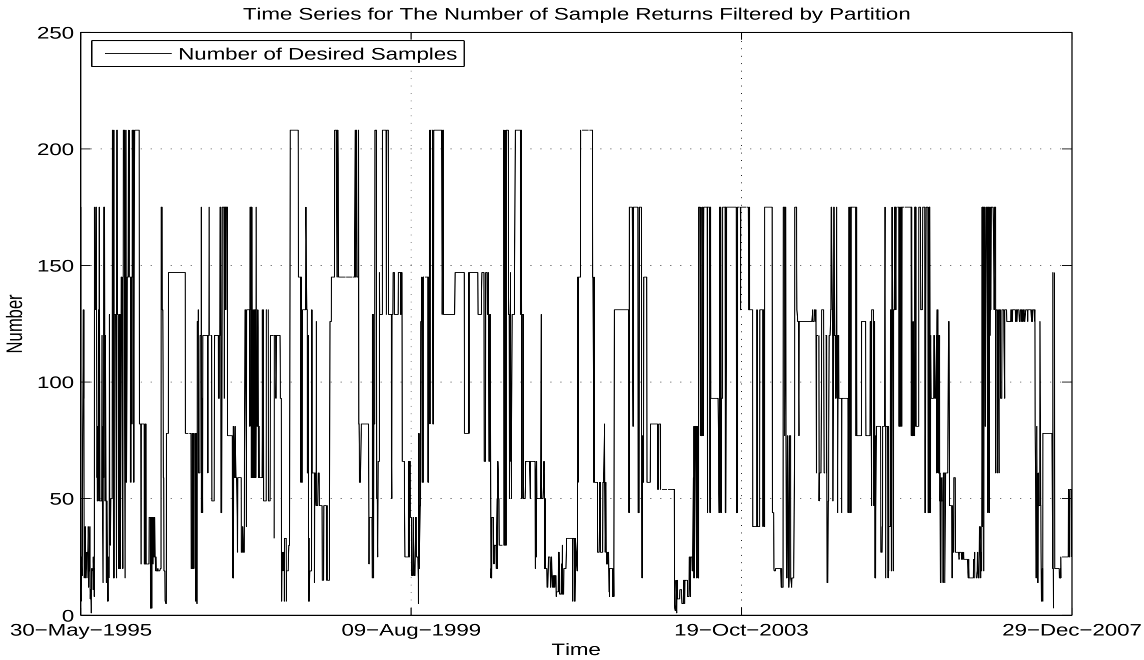

Figure 17: Time series for the number of desired sample returns from the equally weighted portfolio which consists of 213 Russell 2000-listed stocks where $\tau = 100$ and partitions = 3.

Table 6: Duration of investment positions and transaction cost for the strategy without short selling.

Long	2174 days
Risk-free	971 days
Average Long Period	9.49 days
Average Risk-free Period	4.24 days
Number of Long Runs	229
Number of Risk-free Runs	229
Number of Transaction Costs Incurred	458

3.1.4 Simulation for Russell 2000-listed stocks

We perform Monte Carlo simulation for 213 Russell 2000-listed stocks similar to that of S&P 500 listed stocks in Section 3.1.2. The data set is reduced to between 50 and 150 of the 213 stocks which are drawn at random in order to randomize the stocks. Figure 21 shows that the difference between average daily logarithmic returns of the strategy and the proxy converges to a positive value of 1.72E-4 based on 500 iterations. Figure 22 displays that the difference between average daily logarithmic returns per risk in terms of standard deviation for the strategy and the proxy converges to a positive value of 0.068. That is, the strategy outperforms the proxy for 213 Russell 2000-listed stocks.

Figure 18: Time series of mean and variance of historical logarithmic return filtered by partition for the equally weighted portfolio which consists of 213 Russell 2000-listed stocks where $\tau = 100$ and partitions $= 3$.

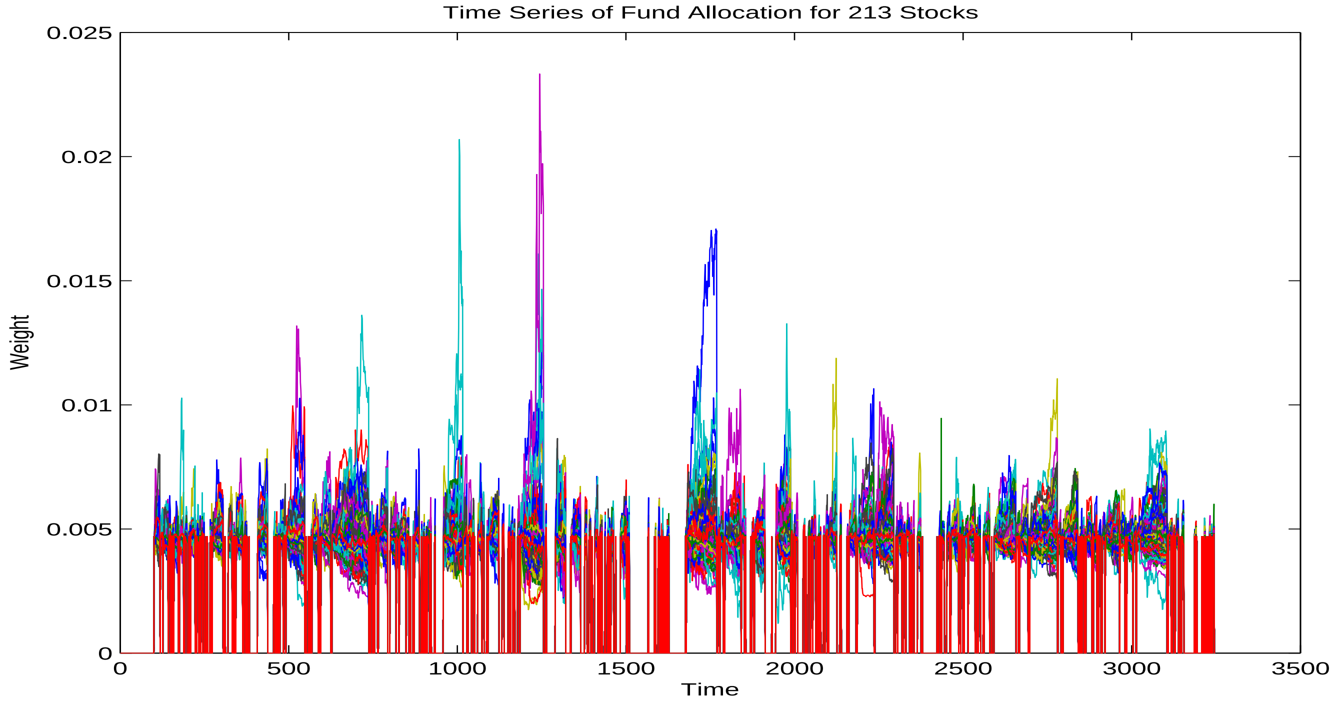

Figure 19: Fund allocation for 213 Russell 2000-listed stocks where $\tau = 100$, partitions = 3, and there is no short selling.

3.2 The use of eigenvalue in addition to the categorization

3.2.1 S&P 500-listed stocks

Figure 23 illustrates the time series of the maximum proportion of the variance via the maximum eigenvalue function. When we compare Figure 23 and Figure 6, we suggest that both the correlation-maximum eigenvalue time series and the standard deviation time series should be considered without replacement.

3.2.2 Simulation for comparison in presence of eigenvalue using S&P 500-listed stocks

Figure 24 shows that the strategy using directional maximum eigenvalue outperforms the strategy using only categorization with the first four moments. The difference of average daily logarithmic returns converges to a positive value of 2.63E-4 for 500 iterations.

3.2.3 Russell 2000-listed stocks

Figure 25 displays the time series of the maximum proportion of the variance via the maximum eigenvalue function for 213 Russell 2000-listed stocks. The convergence diagram of the simulation in Figure 26 shows that the strategy using maximum eigenvalue outperforms the strategy without eigenvalue on average.

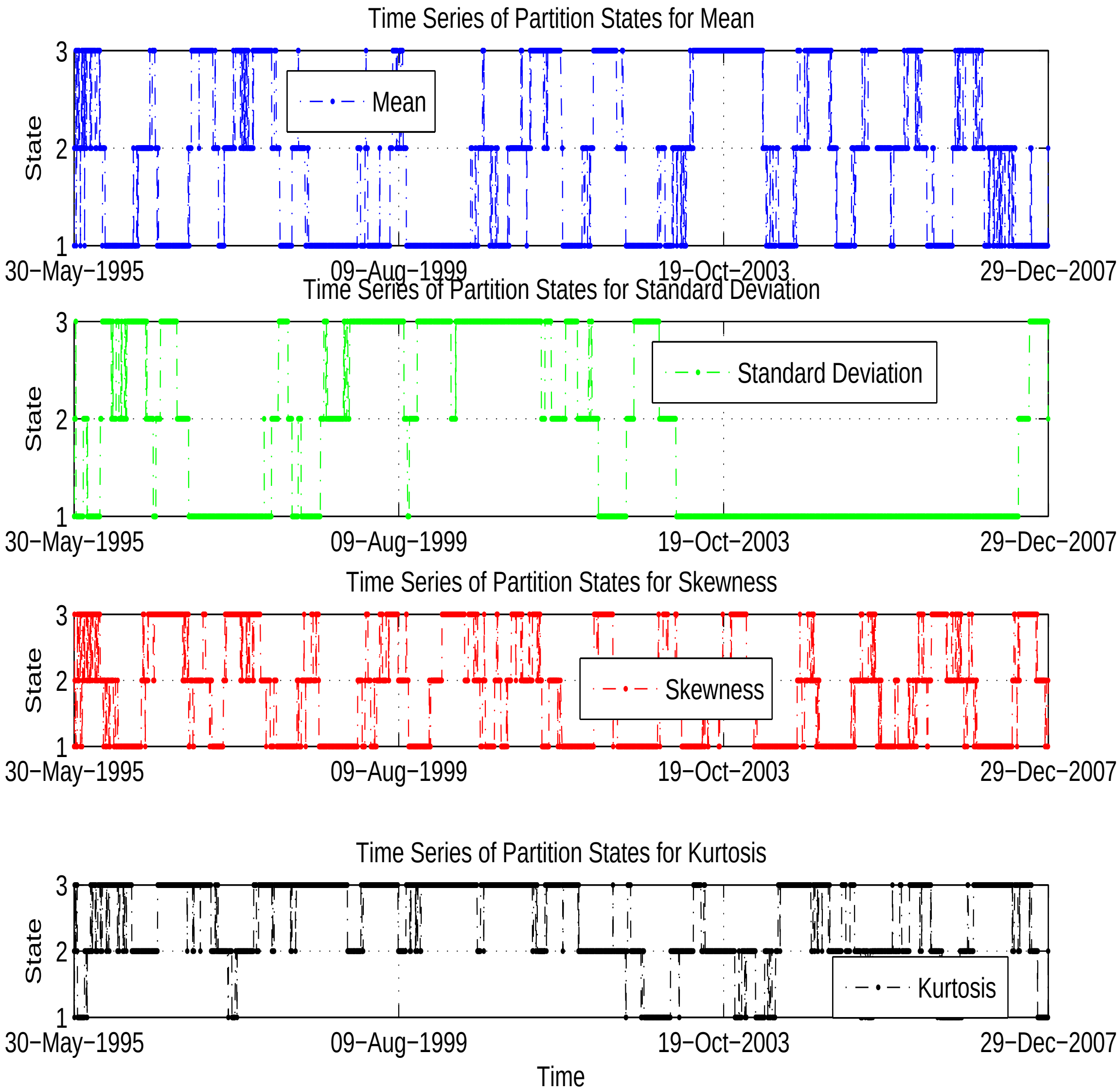

Figure 20: Time-series of partition states for four moments for 213 Russell 2000-listed stocks where $\tau = 100$, partitions $= 3$, and short selling is not allowed.

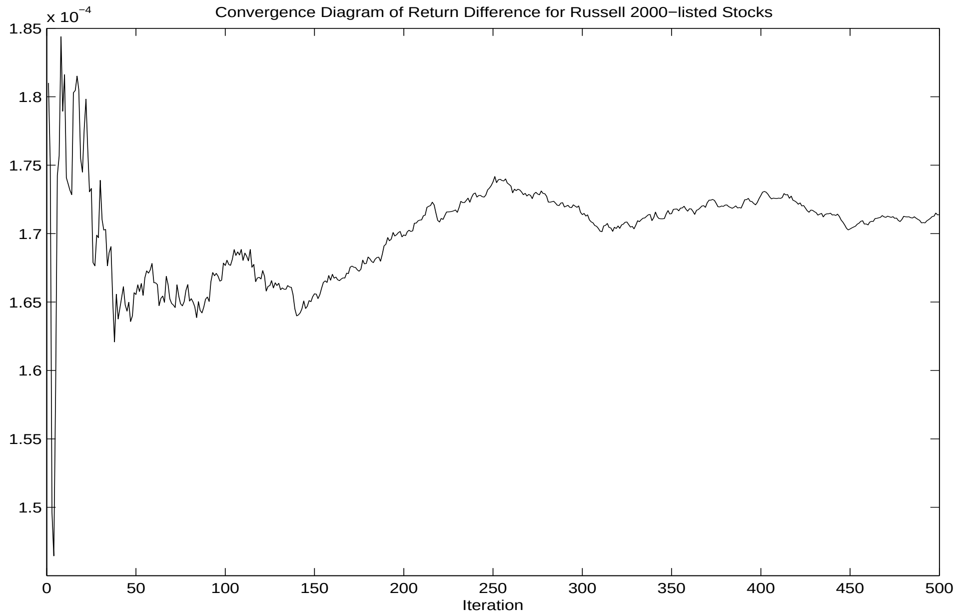

Figure 21: Monte Carlo simulation of return difference for 213 Russell 2000-listed stocks where τ = 100, partitions = 3, and there is no short selling.

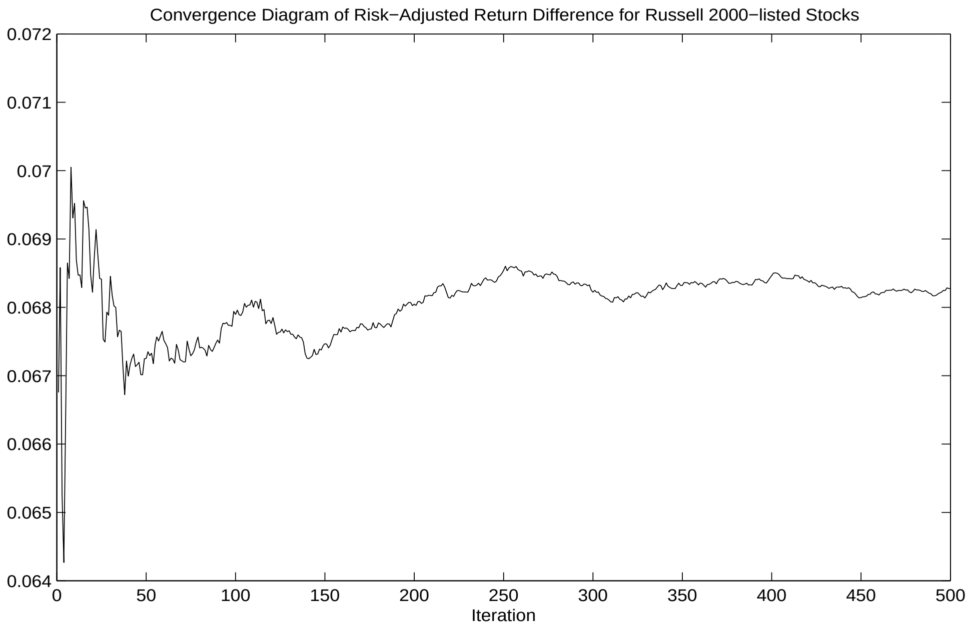

Figure 22: Monte Carlo simulation of risk-adjusted return difference for 213 Russell 2000-listed stocks where τ = 100, partitions = 3, and short selling is not allowed.

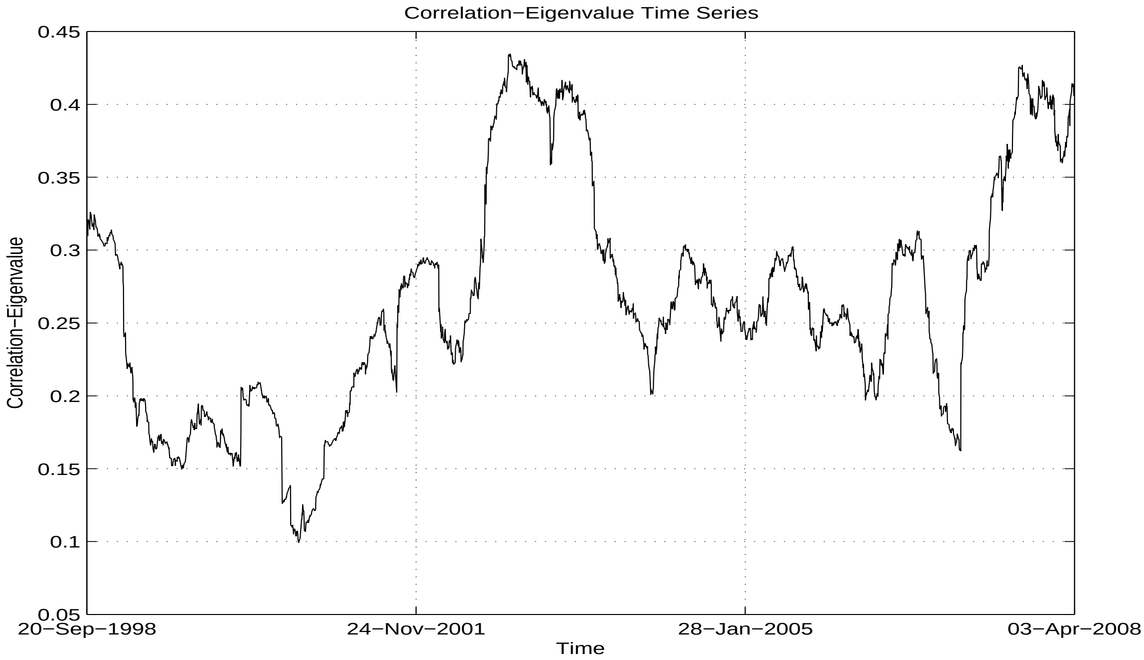

Figure 23: Correlation-maximum eigenvalue time series for 168 S&P 500-listed stocks.

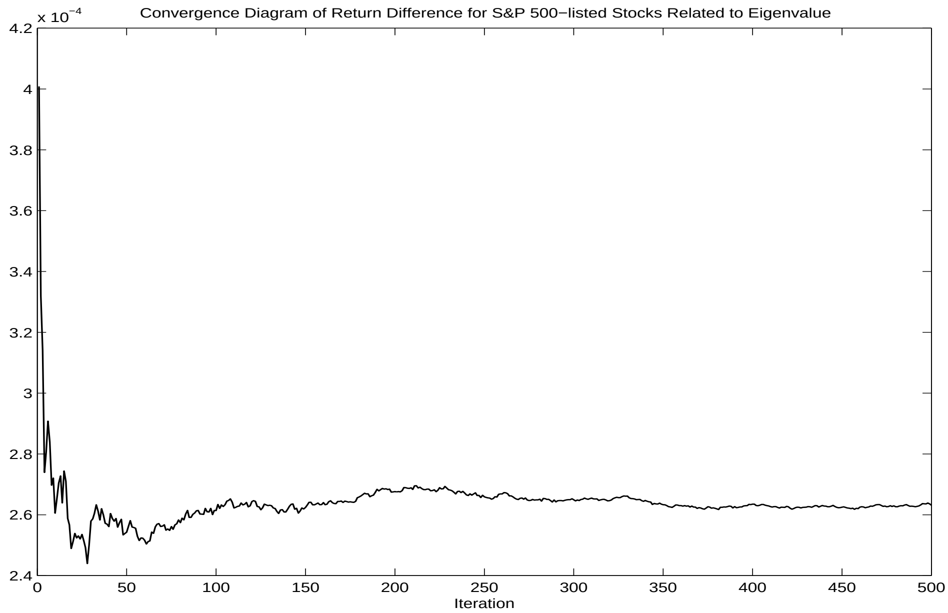

Figure 24: Monte Carlo simulation of return difference for 168 S&P 500-listed stocks where $\tau = 100$, partitions = 3, and short selling is not allowed. It compares the strategy using directional maximum eigenvalue to the strategy using only categorization.

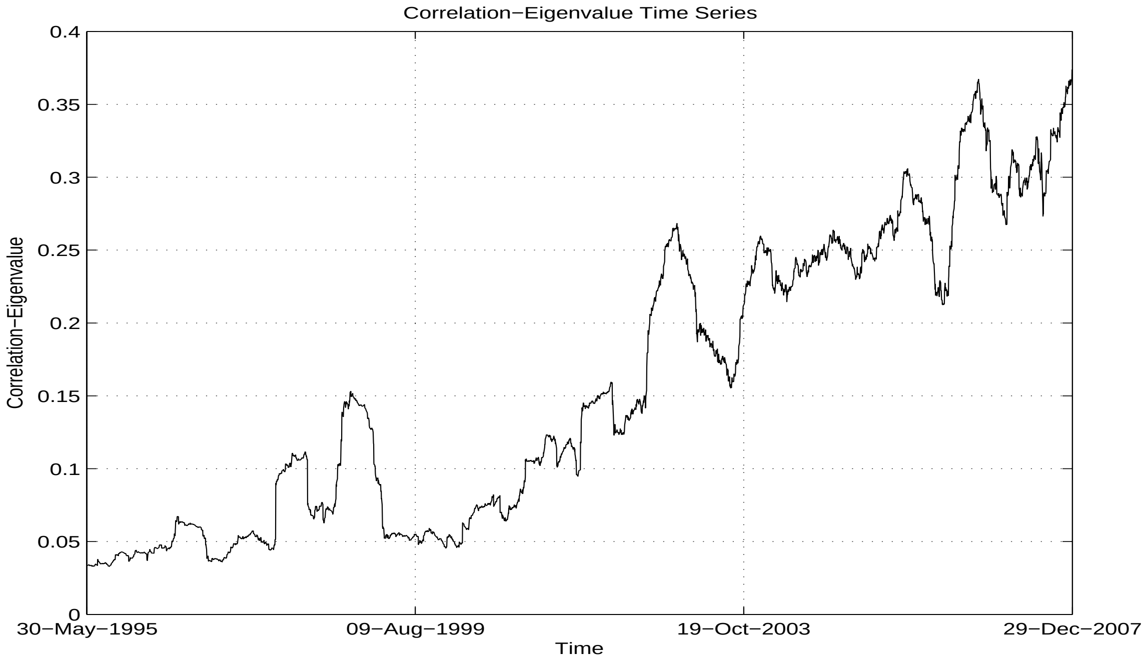

Figure 25: Correlation-maximum eigenvalue time series for 213 Russell 2000-listed stocks.

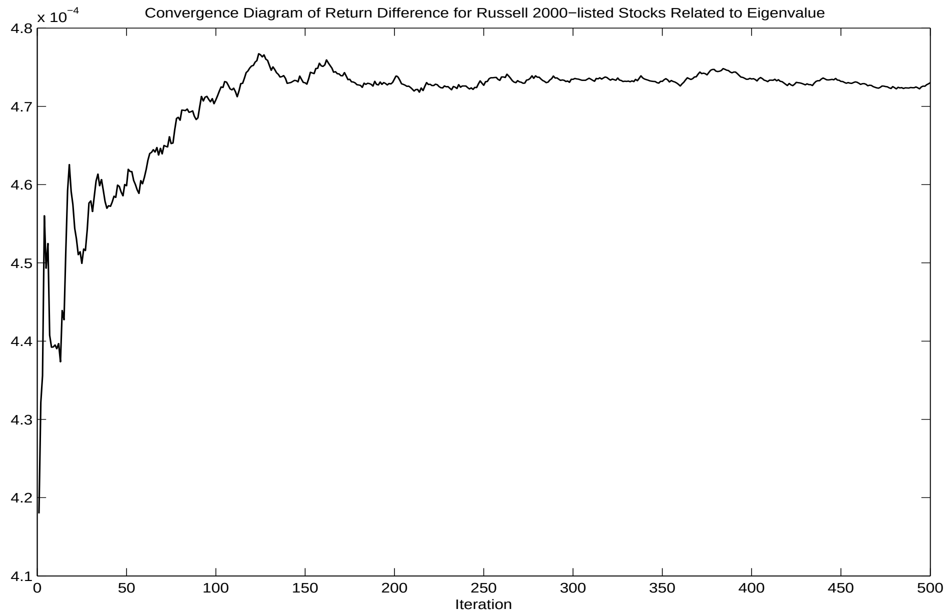

Figure 26: Monte Carlo simulation of return difference for 213 Russell 2000-listed stocks where $\tau = 100$, partitions = 3, and short selling is not allowed. It compares the strategy using directional maximum eigenvalue to the strategy using only categorization.

4 Conclusion

One of the novel components in this paper is the algorithmic trading with the dynamic risk detection based on mean value of historical returns per variance (filtered by partition) and maximum eigenvalue of the daily correlation coefficient matrix. We compare the maximum eigenvalue of recent sample with that of historical data successively. We use such tools for out-of-sample prediction. We experiment the impact of each tool in the algorithm on the performance of strategy incrementally via Monte Carlo simulation over real data. First, we consider the strategy without using maximum eigenvalues. Later, we add the use of two-directional maximum eigenvalue. We find that each tool is needed.

We observe the relationship between long memory and volatility. There may be several explanations for our findings. Quarterly revenue growth and quarterly earnings report may be influential. We believe that long memory (approximately 100 trading days in this study) is a cognitive effect especially related to the behavior of CEO's, analysts, investors and other participants who focus on streaming quarterly goals, announcements and decision making. In addition, we find time intervals where there are persistence in agreement or persistence in disagreement of heterogeneous ad-hoc, structured or other investor groups. This can be explained also by the existence of slowly changing variables in financial markets. Another reason can be the existence of the financial events such as credit crunch which is related to the mortgage problem whose effects may last several months to years.

The investors using our strategy typically, do not change their positions for 4-10 trading days on average (see Table 3 and Table 6). That is, the strategy is lower cost than a daily changing strategy. This may lead the proposed approach to be an appropriate long time investment strategy.

We test our strategy with random initial times and random subsets of stocks via Monte Carlo simulation. The positive return difference with positive probability over finite time intervals indicates the existence of arbitrage opportunity. We use these same random initial times and random subsets for both our strategy and the proxy for the market with the "buy-and-hold" strategy, thus the comparison is fair. To the best of our knowledge, this is the first out-of-sample prediction study to show that there may exist a profitable investment strategy without short selling based on empirical results with daily real risk-free rates, despite the presence of transaction cost.

5 Appendix

Random matrix theory

Marchenko-Pastur formula (see [19]):
The eigenvalue distribution of $N \times N$ random matrix has the spectral density

$$\rho(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{max} - \lambda)(\lambda - \lambda_{min})}}{\lambda}$$

for $\lambda \in [\lambda_{min}, \lambda_{max}]$ where $\lambda_{min}^{max} = 1 + 1/Q \pm 2/\sqrt{Q}$, $Q = (M+1)/N$, and N is the number of time series of length $M + 1$.

Markowitz's optimization scheme

According to classical finance theory (Markowitz [20]) the expected return is

$$\mu_V = mw^T$$

and variance is

$$\sigma_V^2 = wC_{cov}w^T$$

for the return $K_V = \sum_{i=1}^N w_i K_i$ with weights $w = [w_1, ..., w_N]$, expected returns $\mu_i = E(K_i)$ and $m = [\mu_1, \mu_2 ... \mu_N]$. The size of eigenvalue $\lambda_i = \bar{X}_i^T C_{cov} \bar{X}_i$ is a risk measure, when each eigenvector $\bar{X}_i$ is considered as a realization of N security portfolio. The portfolio with the smallest variance in the attainable set has weights

$$w = \frac{uC_{cov}^{-1}}{uC_{cov}^{-1}u^T}.$$

The composition of the least risky portfolio has more weight on the eigenvector of C_{cov} with the smallest eigenvalue, because the smallest λ_i of C_{cov} becomes the largest eigenvalue of C_{cov}^{-1}.

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